Factor Label Method

Would you think I was off my rocker if I wrote mathematical relationships that looked like these:

1 = 24? 1 = 3600? or how about 128 = 3785.4 !

Admit it, you probably would. However, closer analysis might reveal a certain familiarity….

These expressions are incorrect in their present form, but there is something we can do to make them accurate. Do you know what that something is? We can add **UNITS**.

Units are an essential part of any physics problem.

For instance, 1 = 24 clearly is an incorrect mathematical statement.
But what about 1 day = 24 hours! It makes more sense, I know.
Correspondingly, 1 hour = 3600 seconds is ok too.

And, although slightly less commonly known, 128 ounces = 3785.4 milliliters is in fact true. (We can also say 128 ounces = 3785.4 milliliters = 1 gallon and still be correct.)

In your Physics class, ***specifying units is absolutely necessary.***

So what do we do if our answer is 1.24 miles and the question calls for the answer in feet!?

Well we use the FACTOR LABEL method to convert units. It is a series of steps that will help organize even the most challenging conversions (and you will have the chance to try some).

THE METHOD

STEP 1: Start with what you HAVE

In our case we have 1.24 miles

STEP 2: Select an appropriate 'conversion factor' and set up a **multiplication**.

Some may know the conversion factor for feet and miles, that 1 mile = 5280 feet. These two quantities are equal, so we know that 1mile divided by 5280 feet is equal to 1!! So we can multiply what we have by 5280 feet / 1 mile without altering its value. It will look like this:

$$1.24 \text{ miles} \times \frac{5280 \text{ feet}}{1 \text{ mile}} = 6547.2 \text{ feet}$$

notice that the miles on top are divided out by the miles on bottom and we are left with our answer in feet. Also notice that if we turned the fraction over, the units would not divide out.

STEP 3: Continue multiplying by conversion factors if necessary.

The art of the Factor-Label method is to choose your conversion factors wisely, and to set up your fractions in the proper way.

Here is another example:

You know that the speed limit is 65 miles per hour on the highway. But your job is to convert that into meters per second. You know there are 3600 seconds in an hour. You also know that there are 5280 feet in a mile. You are given that 1 meter equals 3.28 feet. How do you proceed?

Step 1: **Start with what you have**. In this case we are given a speed.

$$\frac{65 \text{ miles}}{1 \text{ hour}}$$

Step 2: Select an appropriate conversion factor and set up a multiplication.

$$\frac{65 \text{ miles}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{3600 \text{ seconds}}$$

Step 3: Continue adding factors as necessary. Be sure each factor equals 1!

See that if we perform the multiplication above, we will obtain an answer of ~ .018 miles per second. But that is not in the form we were asked for. So we continue….

$$\frac{65 \text{ miles}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{3600 \text{ seconds}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}}$$

when we perform the multiplication, we will be left with an answer in meters per second!

$$\frac{65 \text{ miles}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{3600 \text{ seconds}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}}$$

see that although the units divide out, we still include the numbers in our calculation.

our final answer is ~ 29.1 meters / second.

On the following pages are some problems for you to solve. Show your work. Set up the fraction multiplications as shown above.

Factor Label Problems

Refer to your book and the whiteboard for the appropriate conversion factors. You will always be provided with the conversion factors you need!

SHOW YOUR WORK

1. 2 liters = how many gallons?

2. 1 year = how many seconds?

3. 19 miles = how many feet?

4. 770 mm Hg = how many Pascals? (note 1 atm = 760 mm Hg and 1 atm also = 1.013×10^2 kPa)

Bonus #6. A light year is how many MILES (light year is a measure of distance, namely how far light would travel in a vacuum over a the course of a year. The value for the speed of light in a vacuum is 3×10^8 m/s)

I know it might seem easy thus far. If you practice, you will get it. But there is a slight twist. What if we measured a rectangular piece of aluminum foil with dimensions of 2 inches by 3 inches. The area then would be 6 in^2 (sometimes referred to as six square inches). What if we needed to convert this into cm^2 ?

It helps to know the conversion factor from inches to cm, that there are 2.54 cm per inch. However we cannot just multiply $6in^2$ x 2.54 cm / inch, as inches does not divide out in^2 ! **This is a crucial distinction.**

Look closely at this approach.

$$6\ in^2 \times \left(\frac{2.54\ cm}{1\ inch}\right)^2$$

see how we square the entire conversion factor? The numbers get squared and the units get squared.

The resulting conversion factor is

$$\frac{2.54^2\ cm^2}{1^2\ inch^2} \quad \text{or} \quad \frac{6.4516\ cm^2}{1\ inch^2}$$

which yields a final answer of

$$6\ in^2 \times \frac{6.4516\ cm^2}{1\ inch^2} = 38.71\ cm^2$$

Good luck with these. No, really. Have fun. Show your work.

Here's another example. Convert 5 cubic yards into cubic feet.

$$5\ yd^3 \times \left(\frac{3\ feet}{1\ yard}\right)^3 = 135\ feet^3$$ (which is the same as 135 cubic feet)

notice how we need to cube the conversion factor to ensure that yd^3 is on the bottom, thus dividing out what we have to begin with.

Factor Label Problems – Part II

SHOW ALL WORK ON SEPARATE PAGES. INCLUDE SKETCHES

1. You are building a deck in your yard. But in order to do so, there is some area under the deck area that you need to fill in with crushed stone to prevent standing water from accumulating under the deck, which may accelerate rotting. Also, some bugs lay eggs in standing water, and who wants mosquitos around when you're eating on the deck? Some people use those spray poisons to kill the bugs but that is no good either. Poisons under the deck!? No good! What about the cat? You measure the dimensions and approximate the volume to be filled in as 270 inches long by 330 inches wide and is 6 inches deep. You find on the internet that you can buy crushed stone for about $15.00 a cubic yard (that's yd^3). How much will you expect to pay for the stone?

2. Your friend boasts to you that he just bought a new motorcycle. He says the engine is "a 1200". You ask him what that means and he doesn't really know. Well, you do some research and find out that it means that the pistons displace 1200 cc of volume during their compression cycle. A cc is short hand for cubic centimeter, or cm^3. How many cubic inches is that? How many cubic meters?

3. The earth has a radius of about 6.38×10^6 meters. Assuming it is a sphere, a) what is its surface area in square miles? By the way, the surface area of a sphere is related to its radius. Surface area = $4\pi r^2$. If the mass of the earth is 5.98×10^{24} kg, b) what is its **average density in kg / m^3** ? Volume of a sphere= $4(\pi r^3) / 3$

4. You are building a pool in your yard next to your deck. Who knew you were such a do-it-yourself kind of person? It is rectangular with dimensions of 18 feet by 36 feet. It is the same depth everywhere, at 5 feet. When it is done, you need to fill it up with water using the hose. Wondering how long this might take, you take an empty milk jug (the one gallon kind), and time how long it takes you to fill it up. You do this a few times and come up with an average time of about 14 seconds. How long will it take you to fill up your pool? Please provide your answer in the units of days, not seconds.